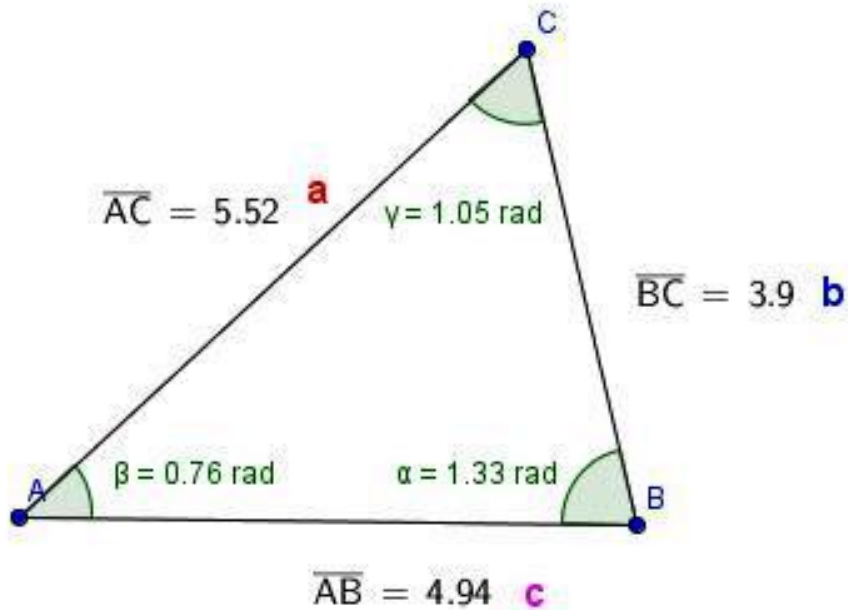


# Teorema dei seni

Con geogebra, derive, cabri, excel



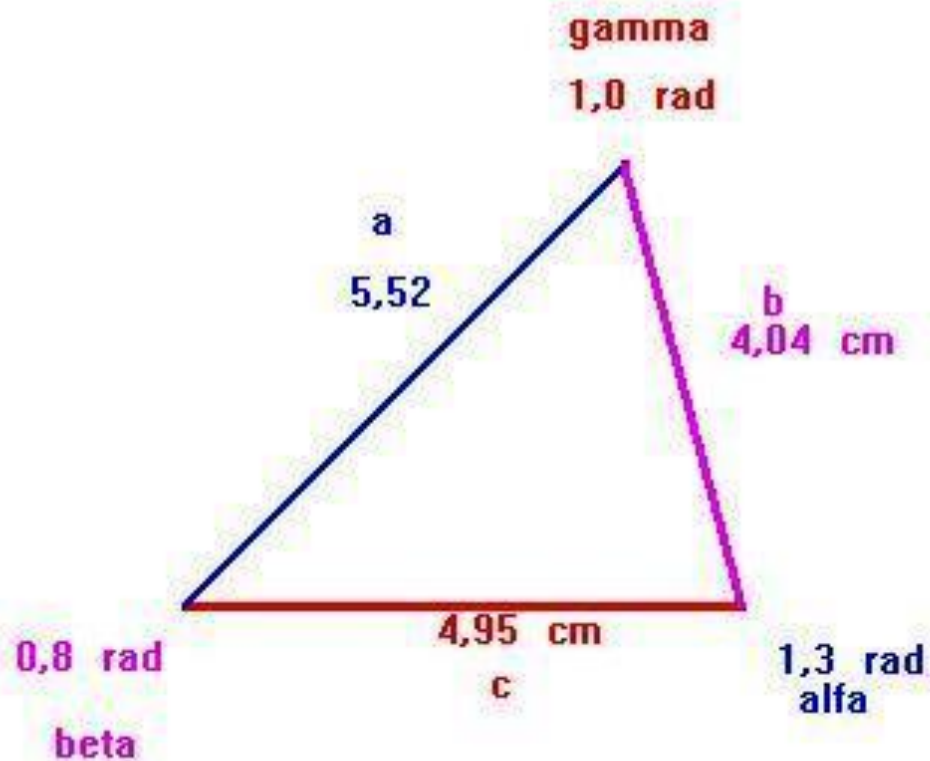
noti lato e due angoli  $a, \beta, \gamma$   
 calcolare  $\alpha, b, c$

noti due lati e angolo tra essi  $a, b, \gamma$   
 calcolare  $c, \beta, \alpha$

noti due lati e angolo opposto  $a, b, \alpha$   
 calcolare  $c, \beta, \gamma$

noti  $a, b, c$   
 calcolare  $\alpha, \beta, \gamma$

eseguire i calcoli, cfr. valori letti con icona



noti a, beta, gamma  
calcolare b, c, alfa

calcolo alfa =  $3,14 - \beta - \gamma$

Risultato:  $1,31 \text{ rad}$

dalla

$$a : \sin(\alpha) = b : \sin(\beta)$$

$$b = a \cdot \sin(\beta) / \sin(\alpha)$$

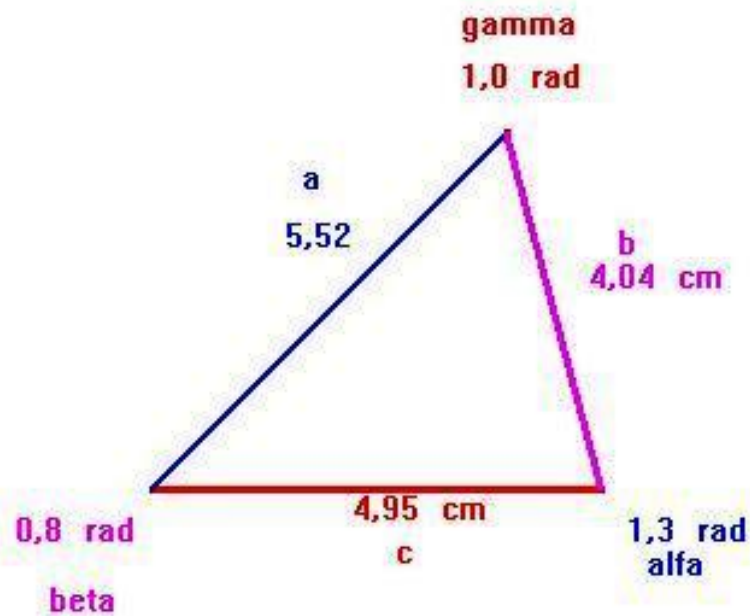
Risultato:  $4,04$

dalla

$$a : \sin(\alpha) = c : \sin(\gamma)$$

$$c = a \cdot \sin(\gamma) / \sin(\alpha)$$

Risultato:  $4,95$



$$a : \sin(\alpha) = b : \sin(\beta) = c : \sin(\gamma)$$

noti  $a$ ,  $b$ ,  $\alpha$   
calcolare  $c$ ,  $\beta$ ,  $\gamma$

dalla  
 $a : \sin(\alpha) = b : \sin(\beta)$  calcolo  $\beta$

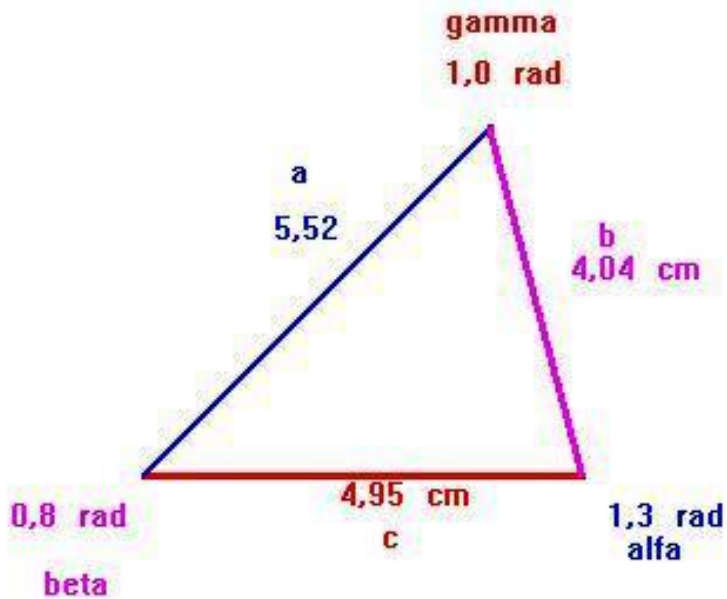
$$\sin(\beta) = b * \sin(\alpha) / a \quad \text{Risultato: 0,71 cm}$$

$$\arcsin(0,71) \quad \text{Risultato: 0,79 rad}$$

$$\text{calcolo } \gamma = 3,14 - \beta - \alpha \quad \text{Risultato: 1,04 rad}$$

dalla  
 $a : \sin(\alpha) = c : \sin(\gamma)$  calcolo  $c$

$$c = a * \sin(\gamma) / \sin(\alpha) \quad \text{Risultato: 4,94}$$



$$a : \sin(\alpha) = b : \sin(\beta) = c : \sin(\gamma)$$

noti  $a, b, \gamma$   
calcolare  $c, \beta, \alpha$

calcolo  $c$  con Carnot

$$c = \sqrt{a^2 + b^2 - 2ab \cos(\gamma)} \quad \text{Risultato: } 4,95$$

dalla

$a : \sin(\alpha) = c : \sin(\gamma)$  calcolo  $\gamma$

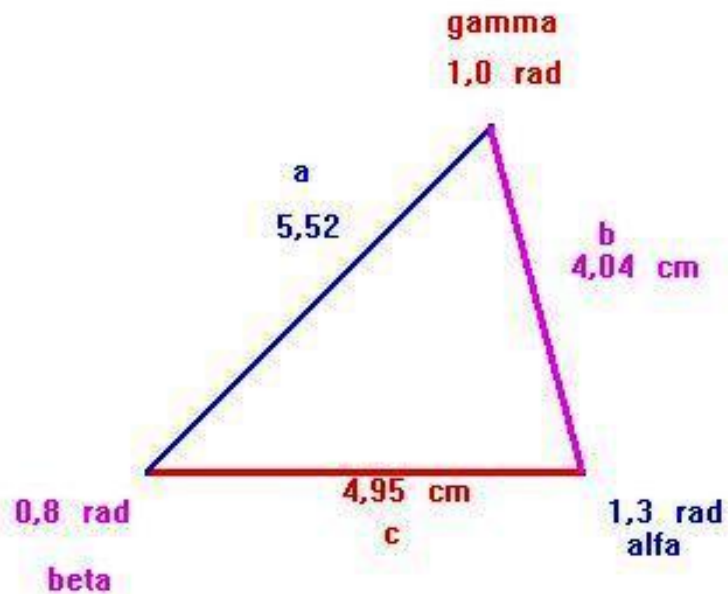
$$\sin(\gamma) = \frac{c \sin(\alpha)}{a} \quad \text{Risultato: } 0,87$$

$$\arcsin(0.79)$$

$$\text{Risultato: } 1,05 \text{ rad}$$

calcolo  $\beta = 3.14 - \alpha - \gamma$

$$\text{Risultato: } 0,79 \text{ rad}$$



$$a : \sin(\alpha) = b : \sin(\beta) = c : \sin(\gamma)$$

noti a, b, c  
calcolare gamma, beta, alfa

calcolo un angolo con Carnot  
posso calcolare altri angoli  
con Carnot, seni, differenza da 3.14

calcolo  $\cos(\alpha)$ , alfa

$$\frac{b^2 + c^2 - a^2}{2bc} \quad \text{Risultato: } 0,26 \quad \text{Risultato: } 1,31 \text{ rad}$$

calcolo  $\sin(\beta)$ ,  $\arcsin(\beta)$ , beta

$$\sin(\beta) = b \cdot \sin(\alpha) / a \quad \text{Risultato: } 0,71 \text{ cm}$$

$$\arcsin(0,71) \quad \text{Risultato: } 0,79 \text{ rad}$$

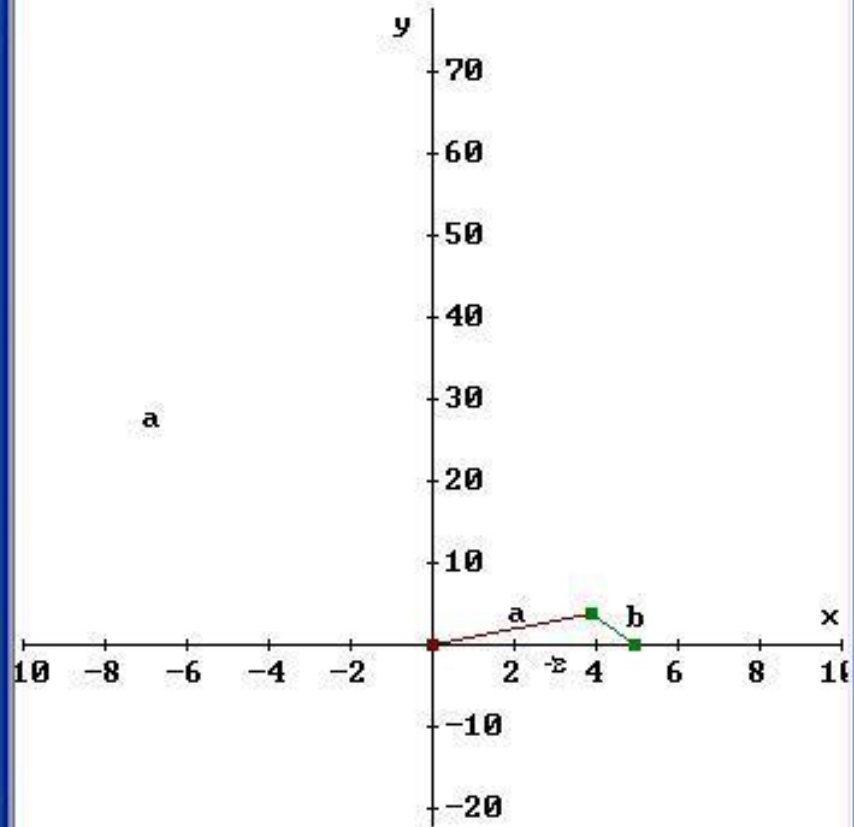
$$\text{calcolo } \gamma = 3,14 - \alpha - \beta \quad \text{Risultato: } 1,05 \text{ rad}$$



Algebra ??? .MTH

```
#1: "teorema dei seni"  
#2: "a : sin(α) = b : sin(β) = c : sin(γ) "  
#3: a := 5.52  
#4: b := 3.9  
#5: c := 4.95  
#6: α := 1.33  
#7: β := 0.76  
#8: γ := 1.05  
#9: "....."  
#10: [ 0 0 ]  
      [ 4.95 0 ]  
#11: [ 0 0 ]  
      [ 3.9 3.9 ]  
#12: [ 3.9 3.9 ]  
      [ 4.95 0 ]
```

Grafici-2D



Algebra ??? .MTH

#14: "noti  $a, \beta, \gamma$  ; calcola  $b, c, \alpha$  "

#15: "calcolo  $\alpha$ "

#16:  $\alpha := \pi - \beta - 1.05$

#17: 1.3

#18: "calcolo  $b, c$ "

#19:  $b := \frac{a \cdot \text{SIN}(\beta)}{\text{SIN}(\alpha)}$

#20: 3.9

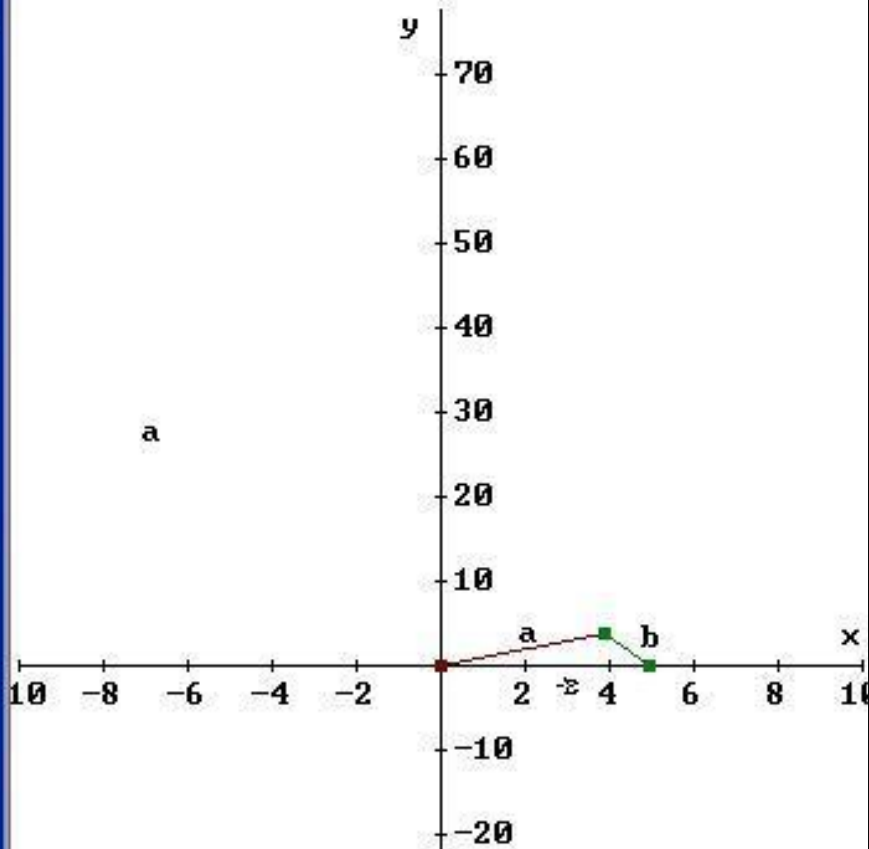
#21: "calcolo  $c$ "

#22:  $c := \frac{a \cdot \text{SIN}(1.05)}{\text{SIN}(\alpha)}$

#23: 4.9

#24: "....."

Grafici-2D





Algebra ??? .MTH

#24: "....."

#25: "a : sin( $\alpha$ ) = b : sin( $\beta$ ) = c : sin( $\gamma$ ) "

#26: "noti a, b ,  $\alpha$  ; calcola c, beta, gamma"

#27: "calcolo sin( $\beta$ ), asin( $\beta$ ), beta"

#28: 
$$\frac{b \cdot \text{SIN}(\alpha)}{a}$$

#29: 0.68

#30: ASIN(0.68)

#31: 0.74

#32: "calcolo gamma"

#33:  $\pi - \alpha - 0.74$

#34: 1.0

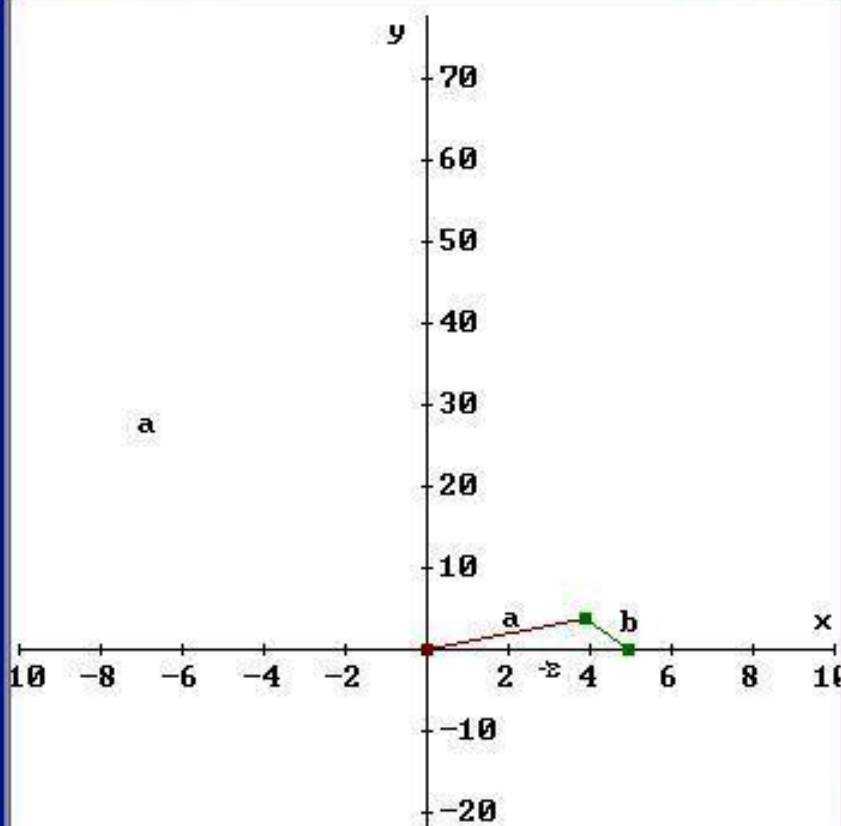
#35: "calcolo c"

#36: 
$$\frac{a \cdot \text{SIN}(1.05)}{\text{SIN}(\alpha)}$$

#37: 4.9

#38: "....."

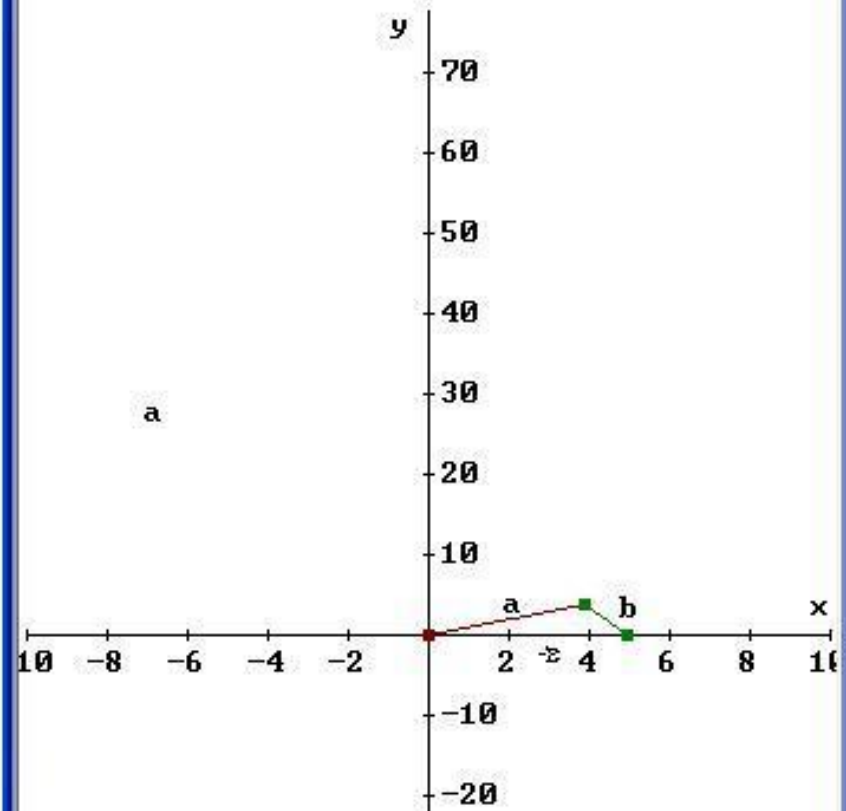
Grafici-2D



Algebra ??? .MTH

```
#38: "....."  
#39: "a : sin(α) = b : sin(β) = c : sin(γ) "  
#40: "noti a, b , γ ; calcola c, α, β "  
#41: "calcolo c con Carnot"  
#42:  $a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos(1.05)$   
#43: 24.3  
#44:  $\sqrt{24.3}$   
#45: 4.9  
#46: "calcolo α"  
#47:  $\frac{a \cdot \sin(1.05)}{c}$   
#48: 0.96  
#49:  $\text{ASIN}(0.96)$   
#50: 1.2  
#51: "calcolo beta"  
#52:  $\pi - \alpha - 1.05$   
#53: 0.76  
#54: "....."
```

Grafici-2D



```

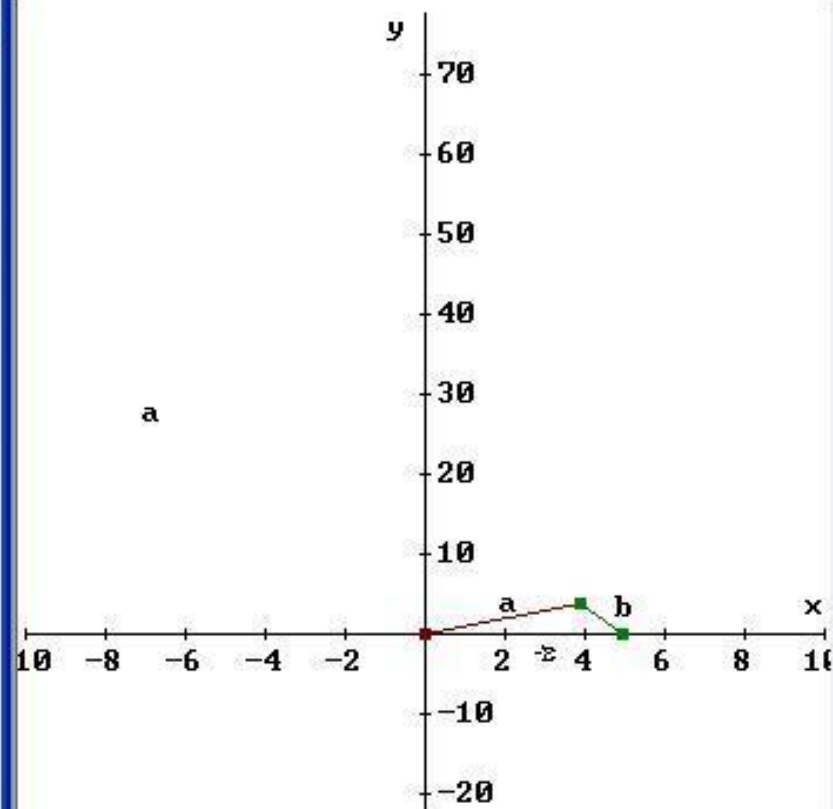
#54: "-----"
#55: "a : sin(α) = b : sin(β) = c : sin(γ) "
#56: "noti a, b , c ; calcola α, β, γ "
#57: "calcolo cos(α), acos(α) , alfa"
#58: 
$$\frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$$

#59: 0.23
#60: ACOS(0.23)
#61: 1.3
#62: "calcolo beta"
#63: 
$$\frac{a^2 + c^2 - b^2}{2 \cdot a \cdot c}$$

#64: 0.72
#65: ACOS(0.72)
#66: 0.76
#67: "calcolo gamma"
#68: 
$$\frac{a^2 + b^2 - c^2}{2 \cdot a \cdot b}$$

#69: 0.49
#70: ACOS(0.49)
#71: 1.0
#72: "possibile usare teorema seni, differenza"
#73: "per calcolare angoli beta, gamma"

```



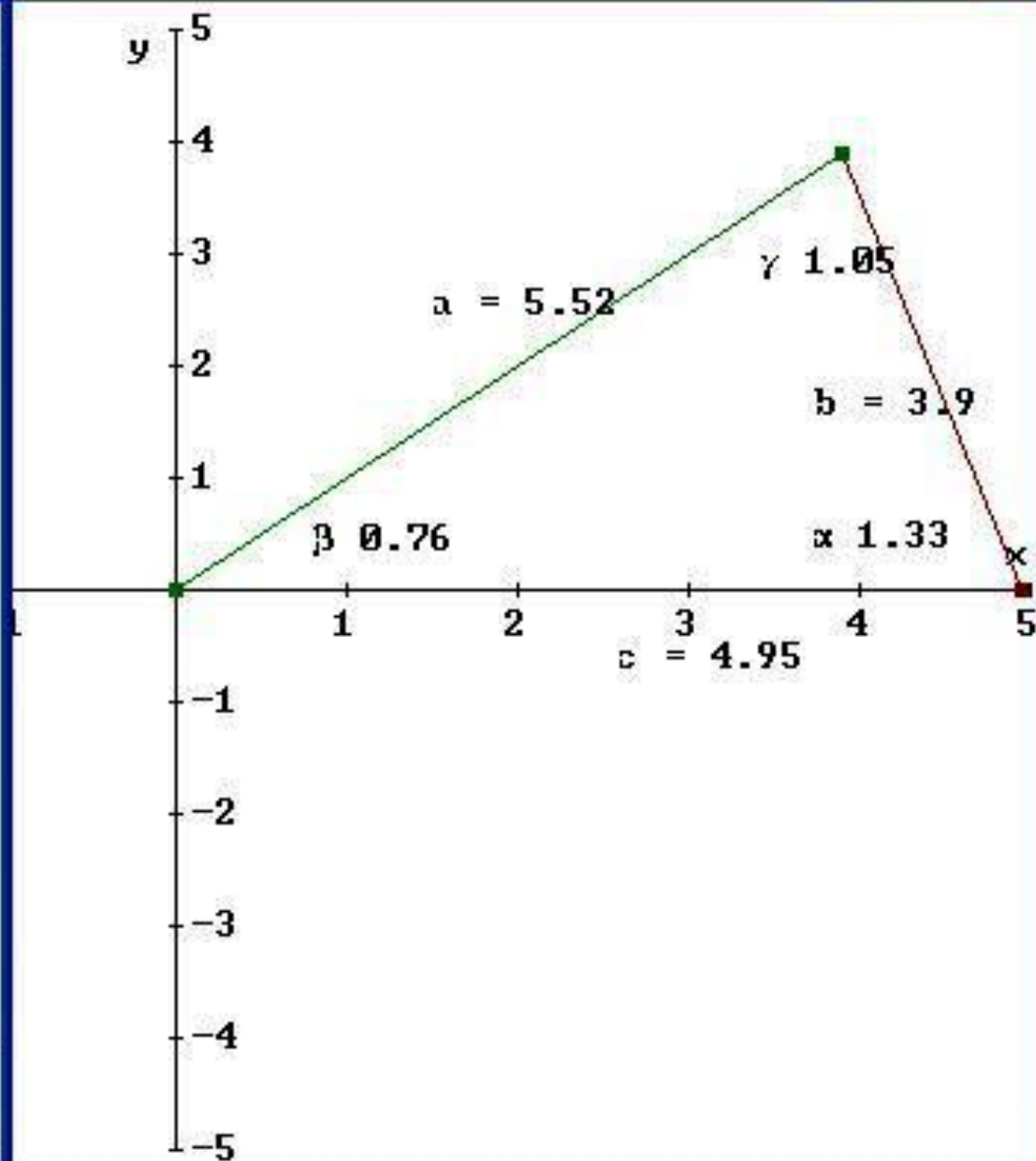


#1: "teorema dei seni"

#2: 
$$\begin{bmatrix} 0 & 0 \\ 3.9 & 3.9 \end{bmatrix}$$

#3: 
$$\begin{bmatrix} 3.9 & 3.9 \\ 4.95 & 0 \end{bmatrix}$$

#4: 
$$\begin{bmatrix} 0 & 0 \\ 4.95 & 0 \end{bmatrix}$$

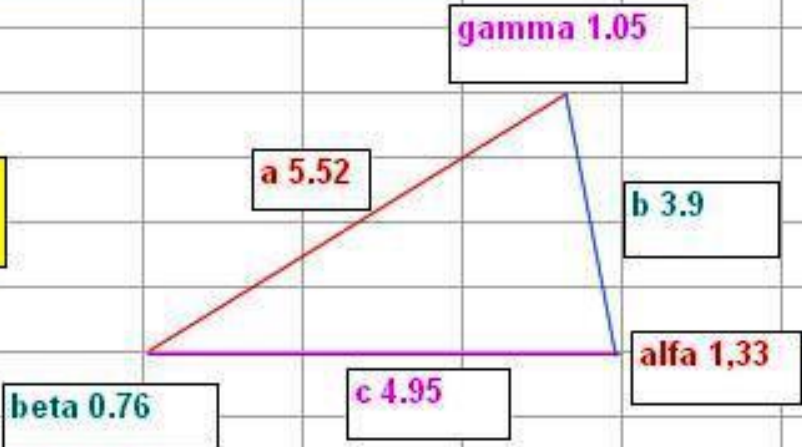


	A	B	C	D	E	F	G
1	a	5,52					
2	beta	0,76	noti a, beta, gamma calcola b, c, alfa				
3	gamma	1,05					
4							
5	alfa	1,33					
6							
7	b	3,915824259					
8							
9	c	4,930427022					
10	teorema dei seni						
11	$a : \sin(\text{alfa}) = b : \sin(\text{beta}) = c : \sin(\text{gamma})$						
12							
13							
14							
15							
16	grafico non in scala valido per numeri						
17							
18							
19							
20							

noti a, beta, gamma  
calcola b, c, alfa

teorema dei seni  
 $a : \sin(\text{alfa}) = b : \sin(\text{beta}) = c : \sin(\text{gamma})$

grafico non in scala  
valido per numeri





	A	B	C	D
1	a	5,52		
2	beta	0,76	noti a, beta, gamma calcola b, c, alfa	
3	gamma	1,05		
4				
5	alfa	=3,14-B2-B3		
6				
7	b	=B1*SEN(B2)/SEN(B5)	grafico non in scala valido per numeri	
8				
9	c	=B1*SEN(B3)/SEN(B5)		
10	teorema dei seni a : sin(alfa) = b : sin(beta) = c : sin(gamma)			
11				
12				
13				

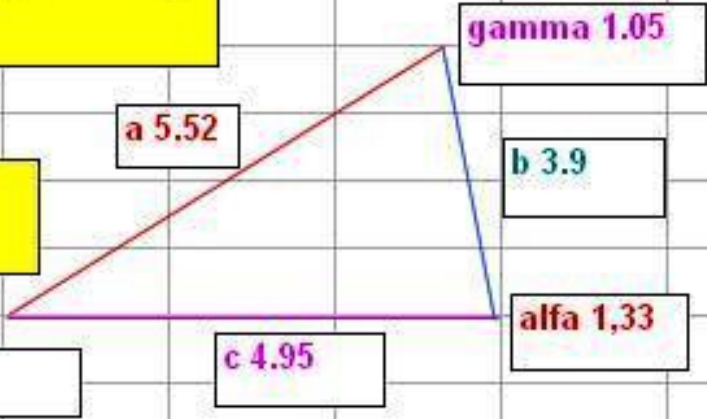


	A	B	C	D	E	F	G
1	a		5,52				
2	b		3,90				
3	alfa		1,33				
4							
5	sen(beta)		0,69				
6	arcsen		0,756166101	beta			
7							
8	gamma		1,05	gamma			
9							
10	c		4,941233754	c			

noti a, b, alfa  
calcola c, beta, gamma

**teorema dei seni**  
 $a : \sin(\text{alfa}) = b : \sin(\text{beta}) = c : \sin(\text{gamma})$

grafico non in scala  
valido per numeri



1	a	5,52
2	b	3,9
3	alfa	1,33

noti a, b, alfa  
calcola c, beta, gamma

5	sen(beta)	=B2*SEN(B3)/B1	
6	arcsen	=ARCSEN(B5)	beta
8	gamma	=3,14-B3-B6	gamma
10	c	=B2*SEN(B8)/SEN(B6)	c

**teorema dei seni**  
 $a : \sin(\text{alfa}) = b : \sin(\text{beta}) = c : \sin(\text{gamma})$

grafico non in scala  
valido per numeri



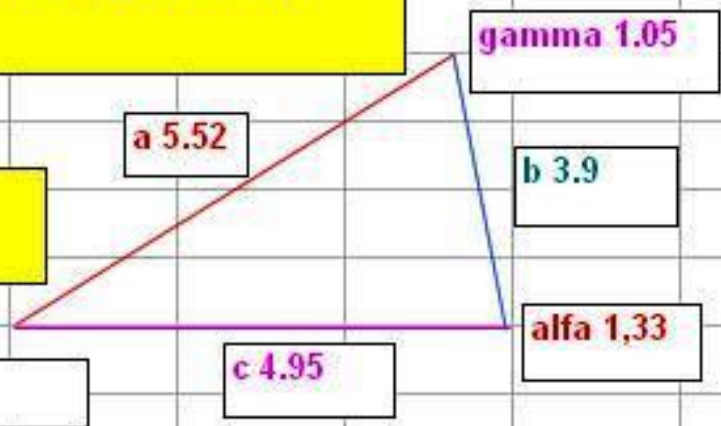
	A	B	C	D	E	F
1	a		5,52			
2	b		3,90			
3	gamma		1,05			

noti a, b, gamma  
calcola c, beta, alfa

6	c		4,925137659	c		
8	cos(beta)		0,73			
9	arccos		0,757179517	beta		
10	alfa		1,36	alfa		

**teorema dei seni**  
 $a : \sin(\text{alfa}) = b : \sin(\text{beta}) = c : \sin(\text{gamma})$

grafico non in scala  
valido per numeri



	A	B	C	D
1	a	5,52		
2	b	3,9		
3	gamma	1,05		
4				
5				
6	c	=RADD(B1^2+B2^2-2*B1*B2*COS(B3))	c	
7				
8	cos(beta)	=(B1^2+B6^2-B2^2)/(2*B1*B6)		
9	arccos	=ARCCOS(B8)	beta	
10	alfa	=3,14-B8-B3	alfa	
11				
12	<b>teorema dei seni</b>			
13	<b>a : sin(alfa) = b : sin(beta) = c : sin(gamma)</b>			
14				
15				

noti a, b, gamma  
calcola c, beta, alfa

**teorema dei seni**  
**a : sin(alfa) = b : sin(beta) = c : sin(gamma)**



A

B

C

D

E

F

G

H

I

a 5,52

b 3,90

c 4,95

noti a, b, c  
calcola gamma, beta, alfa

teorema dei seni

$$a : \sin(\text{alfa}) = b : \sin(\text{beta}) = c : \sin(\text{gamma})$$

cos(alfa) 0,239370629

acos 1,329078743 alfa

beta 0,727618577

acos 0,76 beta

gamma 1,08330268 gamma

grafico non in scala  
valido per numeri

a 5.52

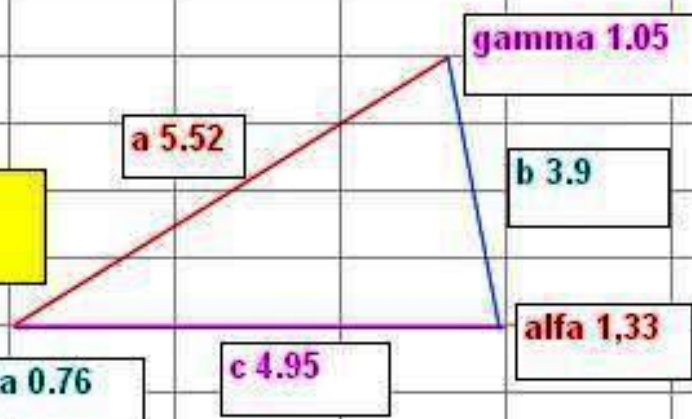
gamma 1.05

b 3.9

alfa 1,33

beta 0.76

c 4.95



	A	B	C	D
1	a	5,52		
2	b	3,9	<div style="border: 1px solid black; padding: 5px;">                     noti a, b, c                      calcola gamma, beta, alfa                 </div>	
3	c	4,95		
4				
5				
6	cos(alfa)	$= (B2^2 + B3^2 - B1^2) / (2 * B2 * B3)$		
7	acos	$= ARCCOS(B6)$	alfa	
8				
9	beta	$= (B1^2 + B3^2 - B2^2) / (2 * B1 * B3)$		
10	acos	$= ARCCOS(B9)$	beta	
11				
12	gamma	$= 3,14 - B7 - B9$	gamma	
13	<div style="border: 1px solid black; padding: 10px;"> <b>teorema dei seni</b>  <math>a : \sin(\text{alfa}) = b : \sin(\text{beta}) = c : \sin(\text{gamma})</math> </div>			
14				
15				
16				

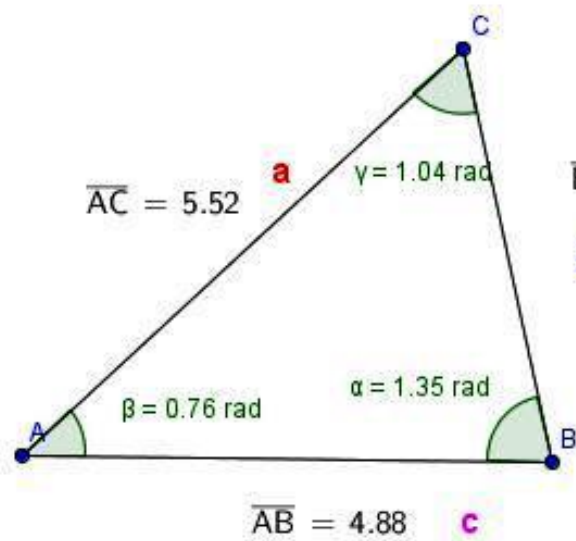




- Vista Algebra
- Angolo
    - $\alpha = 1.35$  rad
    - $\beta = 0.76$  rad
    - $\gamma = 1.04$  rad
  - Numero
    - $d = 1.33$
    - distanzaAB = 4.88
    - distanzaAC = 5.52
    - distanzaBC = 3.89
    - $e = 3.89$
    - $f = 3.9$
    - $g = 4.84$
  - Punto
    - A = (-2.86, 4.14)
    - B = (2.02, 4.08)
    - C = (1.2, 7.88)
    - D = (-0.3, 2.28)
    - E = (1.58, 1.5)
  - Segmento
    - $a = 4.88$
    - $b = 3.89$
    - $c = 5.52$

Vista Grafica

Grid, Zoom, A, G, C, Medio, Pin



noti lato e due angoli  $\alpha, \beta, \gamma$   
calcolare  $\alpha, b, c$

$a = 5.52$   
 $\beta = 0.76$  rad  
 $\gamma = 1.04$  rad

calcolo  $\alpha = \pi - \beta - \gamma$   
 $a / \sin(\alpha) = b / \sin(\beta)$   
calcolo b  
 $b = a * \sin(\beta) / \sin(\alpha)$

$b = 3.9$   
 $c = 4.94$   
 $\alpha = 1.33$  rad

$c / \sin(\gamma) = a / \sin(\alpha)$   
calcolo c  
 $c = a * \sin(\gamma) / \sin(\alpha)$

eseguire i calcoli, cfr. valori letti con icona

Vista Algebra

angolo

- $\alpha = 1.35 \text{ rad}$
- $\beta = 0.76 \text{ rad}$
- $\gamma = 1.04 \text{ rad}$

numero

- $d = 1.33$
- distanzaAB = 4.88
- distanzaAC = 5.52
- distanzaBC = 3.89
- $e = 3.89$
- $f = 3.9$
- $g = 0.69$
- $h = 1.12$
- $i = 5.03$

punto

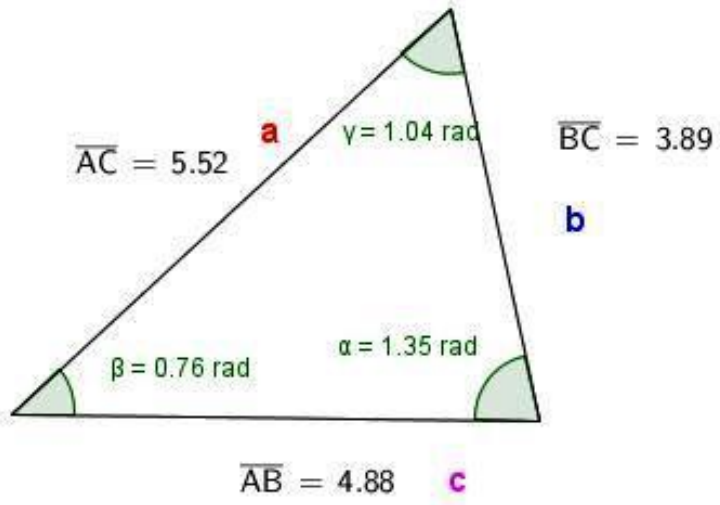
- $A = (-2.86, 4.14)$
- $B = (2.02, 4.08)$
- $C = (1.2, 7.88)$
- $D = (-0.3, 2.28)$
- $E = (1.58, 1.5)$

segmento

- $a = 4.88$
- $b = 3.89$
- $c = 5.52$

Vista Grafica

Grid, Zoom, Color, G, C, Medio, Pointer



$a = 5.52$   
 $b = 3.9$   
 $\alpha = 1.33$

$a : \sin(\alpha) = b : \sin(\beta)$   
 calcolo sin(beta)  
 $\sin(\beta) = b * \sin(\alpha) / a$   
 $3.9 * \sin(1.33) / 5.52$   
 calcolo beta  
 $\beta = 0.69 \text{ radianti (g)}$

calcolo gamma  
 $\pi - 1.33 - 0.69 = 1.12 \text{ rad (h)}$

calcolo v  
 $a : \sin(\alpha) = c : \sin(\gamma)$   
 $c = a * \sin(\gamma) / \sin(\alpha) = 5.03 \text{ (i)}$

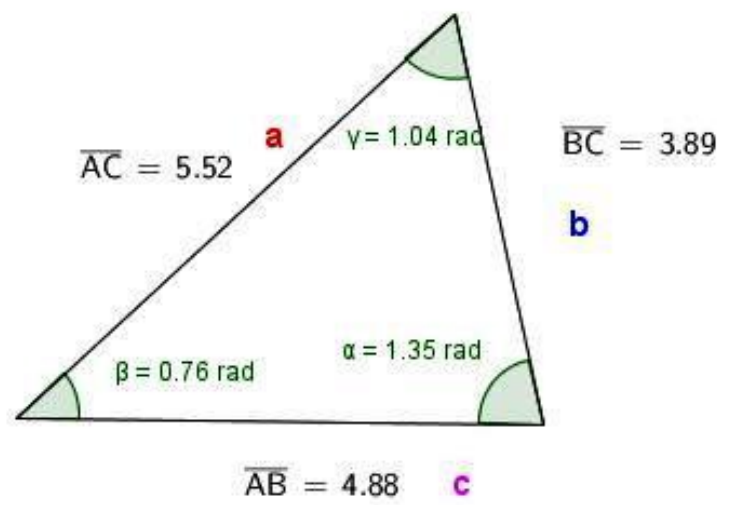
eseguire i calcoli, cfr. valori letti con icona

noti due lati e angolo opposto a, b,  $\alpha$   
 calcolare c,  $\beta$ ,  $\gamma$



Vista Algebra  
 ngolo  
 $\alpha = 1.35 \text{ rad}$   
 $\beta = 0.76 \text{ rad}$   
 $\gamma = 1.04 \text{ rad}$   
 umbero  
 $d = 1.33$   
 distanzaAB = 4.88  
 distanzaAC = 5.52  
 distanzaBC = 3.89  
 $e = 3.89$   
 $f = 3.9$   
 $g = 0.69$   
 $h = 0.76$   
 $i = 4.88$   
 $j = 1.34$   
 unto  
 $A = (-2.86, 4.14)$   
 $B = (2.02, 4.08)$   
 $C = (1.2, 7.88)$   
 $D = (-0.3, 2.28)$   
 $E = (1.58, 1.5)$   
 gmento  
 $a = 4.88$   
 $b = 3.89$   
 $c = 5.52$

Vista Grafica  
 G C Medio



$a = 5.52$   
 $b = 3.89$   
 $\gamma = 1.04$

calcolo lato c con Carnot  
 $c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma)$   
 $c = \sqrt{a^2 + b^2 - 2ab \cdot \cos(\gamma)} = 4.88 \text{ (i)}$

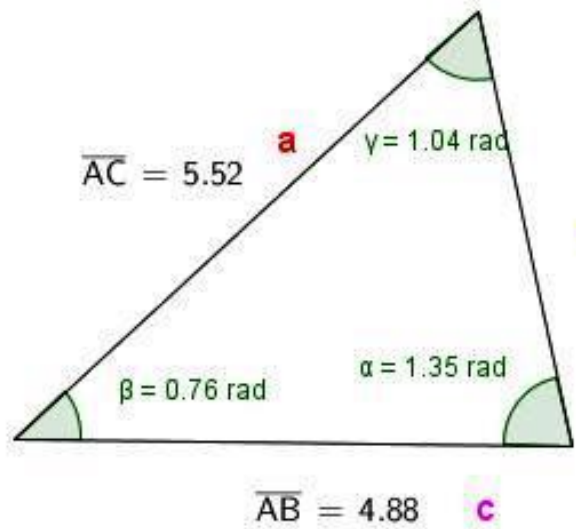
calcolo angolo  $\beta$   
 $c : \sin(\gamma) = b : \sin(\beta)$   
 $\sin(\beta) = b \cdot \sin(\gamma) / c \text{ (0.69 g)}$   
 $\beta = \arcsin(\dots) = 0.76 \text{ (0.76 h)}$

calcolo angolo alfa  
 $\pi - 0.76 - 1.04 = 1.34 \text{ (1.34 j)}$

eseguire i calcoli, cfr. valori letti con icona  
 noti due lati e angolo tra essi a , b ,  $\gamma$   
 calcolare c ,  $\beta$  ,  $\alpha$



- Angolo
  - $\alpha = 1.35 \text{ rad}$
  - $\beta = 0.76 \text{ rad}$
  - $\gamma = 1.04 \text{ rad}$
- Numero
  - $d = 1.33$
  - distanzaAB = 4.88
  - distanzaAC = 5.52
  - distanzaBC = 3.89
  - $e = 0.22$
  - $f = 1.35$
  - $g = 0.73$
  - $h = 0.75$
  - $i = 0.93$
- Punto
  - $A = (-2.86, 4.14)$
  - $B = (2.02, 4.08)$
  - $C = (1.2, 7.88)$
  - $D = (-0.3, 2.28)$
  - $E = (1.58, 1.5)$
- Segmento
  - $a = 4.88$
  - $b = 3.89$
  - $c = 5.52$



$a = 5.52$   
 $b = 3.89$   
 $c = 4.88$

applico Carnot per calcolare un angolo  
 posso calcolare altro angolo con Carnot  
 oppure con teorema dei seni  
 posso calcolare terzo angolo con Carnot,  
 teorema dei seni, differenza

eseguire i calcoli, cfr. valori letti con icona  
 noti a, b, c  
 calcolare  $\alpha, \beta, \gamma$

calcolo angolo  $\alpha$  dalla relazione  
 $a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$   
 $\cos(\alpha) = (b^2 + c^2 - a^2) / 2bc$   
 $\cos(\alpha) = 0.22$  (e)  
 calcolo  $\alpha$  con  $\text{acos}(\alpha)$   
 $\text{alfa} = 1.35$  (f)

$b^2 = a^2 + c^2 - 2ac \cdot \cos(\beta)$   
 $\cos(\beta) = (a^2 + c^2 - b^2) / (2 \cdot a \cdot c)$   
 $\cos(\beta) = 0.73$  (g)  
 $\text{acos}(0.73) = 0.75$  (h)

calcolo  $\gamma = 3.14 - 0.76 - 1.35 = 0.93$  (i)



Vista Algebra

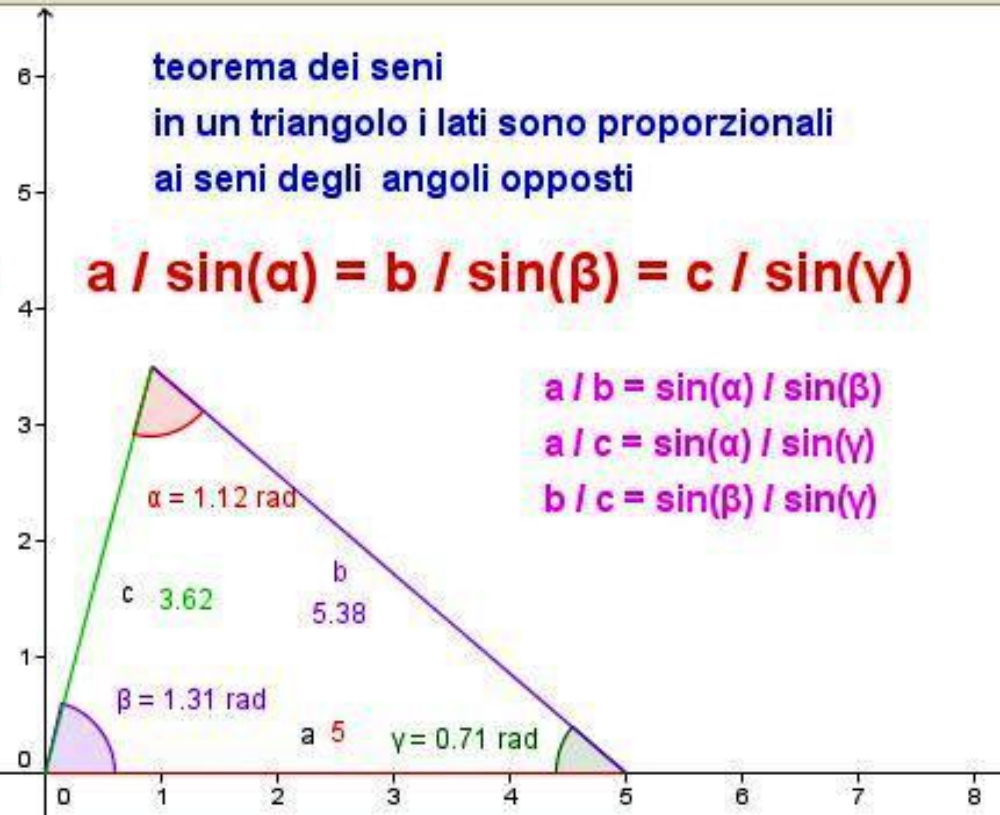
Vista Grafica

- Angolo
  - $\alpha = 1.12 \text{ rad}$
  - $\beta = 1.31 \text{ rad}$
  - $\gamma = 0.71 \text{ rad}$
- Numero
  - $d = 5.55$
  - $e = 5.57$
  - $f = 5.55$
- Punto
  - $A = (0, 0)$
  - $B = (5, 0)$
  - $C = (0.92, 3.5)$
- Segmento
  - $a = 5$
  - $b = 5.38$
  - $c = 3.62$

d.....e.....f = 5.55  
rapporto lato/seno = costante

teorema dei seni  
in un triangolo i lati sono proporzionali  
ai seni degli angoli opposti

$$a / \sin(\alpha) = b / \sin(\beta) = c / \sin(\gamma)$$



- 4 tipi di problemi in funzione degli elementi noti
- lato e due angoli
  - due lati e angolo tra essi
  - due lati e angolo opposto a un lato
  - tre lati (Carnot)